Modification of spin mixing of spinor BEC by cavity QED coupling

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Dressed states of spinor Bose-Einstein condensates of spin-1 atoms coupling with optical cavity modes with far off resonance frequency are investigated. The exact solution of time evolution of population of spin component is derived, and the numerical result shows that the evolution is different from spin mixing without the coupling. Due to the coupling with the atoms, the photon state also evolute to different optical cavity modes.

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I. INTRODUCTION

Cavity Quantum Electrodynamics (cavity QED) [1, 2] describes the coupling between atom and confined cavity mode of electromagnetic field by coherent interaction. It gives a great chance to reveal the nature of coherent interaction between matter and field, and to various application such as quantum information and quantum state engineering. The atom can be described by two-level system, and the cavity mode is described as quantized light field. The coupling between two-level system and quantized light field is described by the Jaynes-Cummings model [3], which describe the coupling strength as the speed of energy transfer between two subsystem (here the excitation energy of the atom is close to energy level of cavity mode, which is called near resonance). When the energy level distribution of the atom is more complicated, three-level system or other multi-level system is needed to describe the atom. But the description of coupling is similar. Some multi-level system give interesting phenomenon such as electromagnetical induced transparent [4].

When there are more than one atom, the system can be described by Tavis-Cummings model [5], where N atoms are assumed to couple to the same cavity mode with identical coupling strength. This lead to enhance of the coupling strength to $\sqrt{N}$ time. When the temperature of the system is lower than critical temperature, the N atoms couple with each other coherently and form Bose-Einstein condensate (BEC). In this case, all atoms have the same quantum state, which ensure the same coupling strength with cavity mode. As a result, the Tavis-Cummings model is very appropriate for this system. A simpler model is to describe the BEC of the N atoms by mean field theory. A dressed BEC state [6] and a dark state [7] is investigated by the mean field theory description. When the energy level of cavity mode is much different from excitation energy of the atoms (far off resonance), the coupling can be described by effective Hamiltonian [6], which is given by solving the Schrodinger equation of single atom and cavity mode. The dynamic property of dress BEC state of far off resonance is also investigated under mean field theory method [6].

When the N atoms is trapped in an total optical trap, the spin freedom of the atoms is librated. When temperature is low enough, spinor BEC is form [8, 9], which has much potential in application of quantum computer and quantum information [10]. Spinor BEC can be described by either mean field theory or analytical express of the quantum state. A former work about fractional parentage coefficients (FPCs) gives a chance to describe spinor BEC of spin-1 atoms in a simpler way [11, 12]. One of the most interesting phenomenon of spinor BEC is spin mixing, where the average atomic number of each spin component evolute versus time [13, 14]. The pattern of time evolution is found to be decided by symmetric property and initial condition of the system [15]. If the spinor BEC is placed inside a far off resonant cavity, the atoms will couple to the cavity mode, which change the symmetric of the system as well as the effective coupling strength between atoms. As a result, the evolution of spin component is supposed to be modified. In this paper, we will deduce the effective Hamiltonian to describe the far off resonant coupling of spinor BEC and cavity mode, and figure the dressed state of spinor BEC which is eigenstate of the Hamiltonian. And then we investigate the modification of spin mixing by the coupling. The spinor BEC system is described by FPCs, which can give an analytical solution to the problem [12].

II. DESCRIPTION OF DRESSED SPINOR BEC STATE

The system we are going to investigate consist of N ^{87}Rb atoms which are optically trapped to form a spinor BEC, and located inside a high-Q optical cavity. Assume that there are only two optical modes of frequency ω_c, which are left and right circular polarization optical mode in the cavity. The two optical modes have opposite angular momentum $\hbar$ along $\hat{z}$ direction, which are indexed by $\mu = \pm 1$. The ground state of ^{87}Rb is $5^2S_{1/2}, F = 1$, and the lowest energy state of excited

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state of $D1$ line is $5^2P_{1/2}, F = 1$ [17]. When ω_c is smaller than and at the same scale of the frequency of $5^2S_{1/2}, F = 1 \longrightarrow 5^2P_{1/2}, F = 1$ transition ω_e, we can neglect other transition. As a result, the non-coupling Hamiltonian of the N atoms with spin dependent interaction with the optical frequency removed is

$$\begin{aligned} H_0 &= \sum_{i=1}^{N}(\frac{\mathbf{p}_i^2}{2m} + U(\mathbf{r}_i) + \sum_{\sigma=1,0,-1} \hbar\delta|e_i,\sigma\rangle\langle e_i,\sigma|) \\ &+ \sum_{i\neq j} V_{i,j} \end{aligned} \tag{1}$$

where $\mathbf{p}_i$ is momentum of the ith atom, m is atomic mass, $U(\mathbf{r}_i)$ is potential of external optical trap; $\delta = \omega_e - \omega_c$, $|e_i, \sigma\rangle$ is the $5^2P_{1/2}, F = 1$ state of the ith atom with spin component σ ($|g_i, \sigma\rangle$ is the $5^2S_{1/2}, F = 1$ state of the ith atom with spin component σ); $V_{i,j} = \delta(\mathbf{r}_i - \mathbf{r}_j)(c_0 + c_2\mathbf{F}_i \cdot \mathbf{F}_j)$ is spin dependent interaction between atoms, with $\mathbf{F}$ being spin operator and $c_{0,2}$ being interaction constant [9].

Since the Q factor of the optical cavity is very high, and temperature is very close to zero, the decay of photon and non-coherent interaction between atoms and photon can be neglected. According to the selection rule, if the atom absorb (emit) one photon from(to) the cavity, the allowed transitions have $\Delta\sigma = \pm 1$. Therefore, the Hamiltonian of coupling between atom and cavity mode is given by

$$\begin{aligned} H_C = \sum_{i=1}^{N} \hbar\Omega_0(&|e_i, 0\rangle\langle g_i, -1|\hat{a}_1 + |e_i, 1\rangle\langle g_i, 0|\hat{a}_1 \\ &+|e_i, -1\rangle\langle g_i, 0|\hat{a}_{-1} + |e_i, 0\rangle\langle g_i, 1|\hat{a}_{-1} + c.c) \end{aligned} \tag{2}$$

where $\hat{a}_\mu$ and $\hat{a}_\mu^+$ are annihilation and creation operators of optical modes of polarization $\mu = \pm 1$, $\Omega_0 = dE/\hbar$ is the strength of the atom-field coupling. d is the atomic dipole-matrix element, and $E = \sqrt{\hbar\omega_c/2\epsilon_0 V}$ is electric field of single photon state, V being mode volume. In far off resonance region, we have $\delta >> \Omega_0$.

Using similar process in reference [6], we can get the effective Hamiltonian of the coupling system without involving the atomic excited state

$$\begin{aligned} H_{eff} = & \sum_{i=1}^{N}[\frac{\mathbf{p}_i^2}{2m} + U(\mathbf{r}_i) \\ + & \frac{\hbar\Omega_0^2}{\delta}(-|g_i,-1\rangle\langle g_i,-1|a_1^+a_1 - |g_i,1\rangle\langle g_i,1|a_{-1}^+a_{-1} \\ & +|g_i,-1\rangle\langle g_i,1|a_1^+a_{-1} + |g_i,1\rangle\langle g_i,-1|a_{-1}^+a_1)] \\ + & \sum_{i\neq j} V_{i,j} \end{aligned} \tag{3}$$

It is obvious that the last two single particle terms are given by the process that the atom absorb one photon from one mode and emit one photon to the another mode. Note that the Hamiltonian (3) conserve the total angular momentum at z direction $\mathcal{M}\hbar$ and total number of photons $n = n_1 + n_{-1}$. In order to obtain the eigen state of the Hamiltonian, we first give the eigen state of the Hamiltonian without coupling between atom and optical mode ($\Omega_0 = 0$), which is direct product of eigen state of the N atoms and eigen state of optical mode. And then calculate the matrix element of the Hamiltonian with coupling ($\Omega_0 \neq 0$) under the basic states given by the direct product states. The eigenstate of the Hamiltonian is obtained by diagonalizing the matrix.

The direct product states is given as

$$|S, M; n_1, n_{-1}\rangle = |S, M\rangle_N \otimes |n_1, n_{-1}\rangle \tag{4}$$

where $|S, M\rangle_N$ is eigenstate of the N atoms with total spin S and z component of total spin M, $|n_1, n_{-1}\rangle$ is optical state with n_1 (n_{-1}) photons in $\mu = 1(-1)$ polarization state. The z direction angular momentum of the whole system is $\mathcal{M}\hbar = (M + n_1 - n_{-1})\hbar$, and eigen energy of the states is $E_{S,n_1,n_{-1}} = S(S+1)J$, where J is a constant decided by spin dependent interaction c_2. The atomic eigenstate is given as

$$|S, M\rangle_N = \vartheta_{S,M}^N \prod_{i=1}^{N} \phi(\mathbf{r}_i) \tag{5}$$

where $\vartheta_{S,M}^N$ is the total symmetric spin state of N atoms, $\phi(\mathbf{r})$ is the single particle spatial wave function. Note that we use second quantized Hamiltonian to describe optical mode, while use first quantized Hamiltonian to describe the N atoms. The total symmetric property of the N Bosonic atoms is included in the total symmetric atomic wave function. The total symmetric spin state can be expanded as

$$\vartheta_{S,M}^N = a_S^N[\chi(i)\vartheta_{S+1}^{N-1}]_{S,M} + b_S^N[\chi(i)\vartheta_{S-1}^{N-1}]_{S,M} \tag{6}$$

where $a_S^N = \{[1 + (-1)^{N-S}](N - S)(S + 1)/[2N(2S + 1)]\}^{1/2}$ and $b_S^N = \{[1 + (-1)^{N-S}]S(N + S + 1)/[2N(2S + 1)]\}^{1/2}$ are FPCs given in reference [12], $\chi(i)$ is spin state of the ith spin-1 ground state atom, which is couple to total symmetric spin state of the other $N - 1$ atoms by Clebsch-Gordan coefficient.

The eigenstates with quantum number $(\mathcal{M}\hbar, n)$ can be expanded by the direct product states (4) with the same value of $\mathcal{M}\hbar$ and $n = n_1 + n_{-1}$. Since we are interested in the quantum effect of the system, we focus on one photon case, i.e. $n_1 + n_{-1} = 1$, but the extension to more photon case is straight forward. When there is only one photon, only the direct product states with quantum number $(M = \mathcal{M} - 1, n_1 = 1, n_{-1} = 0)$ or $(M = \mathcal{M} + 1, n_1 = 0, n_{-1} = 1)$ can be basic states. As a result, the eigenstate can be expressed as

$$\begin{aligned} |\mathcal{M}, \nu\rangle = & \sum_{S>=\mathcal{M}-1} c_{S,\mathcal{M}-1}^{\nu}|S, \mathcal{M}-1; 1, 0\rangle \\ + & \sum_{S>=\mathcal{M}+1} c_{S,\mathcal{M}+1}^{\nu}|S, \mathcal{M}+1; 0, 1\rangle \end{aligned} \tag{7}$$

where ν is the index of eigenstate, $c^{\nu}_{S,\mathcal{M}\pm 1}$ is expansion coefficients, and the summation satisfy $N-S$ is even. Making use of equation (6), the matrix elements are obtained as

$$\langle S,\mathcal{M}-\mu;\{\mu\}|H_{eff}|S,\mathcal{M}-\mu;\{\mu\}\rangle = S(S+1)J - \frac{\hbar\Omega_0^2 N}{\delta}[(A^N_{S,\mathcal{M}-\mu,-\mu})^2+(B^N_{S,\mathcal{M}-\mu,-\mu})^2] \quad (8)$$

$$\langle S+2,\mathcal{M}-\mu;\{\mu\}|H_{eff}|S,\mathcal{M}-\mu;\{\mu\}\rangle = -\frac{\hbar\Omega_0^2 N}{\delta}A^N_{S,\mathcal{M}-\mu,-\mu}B^N_{S+2,\mathcal{M}-\mu,-\mu} \quad (9)$$

$$\langle S-2,\mathcal{M}-\mu;\{\mu\}|H_{eff}|S,\mathcal{M}-\mu;\{\mu\}\rangle = -\frac{\hbar\Omega_0^2 N}{\delta}A^N_{S-2,\mathcal{M}-\mu,-\mu}B^N_{S,\mathcal{M}-\mu,-\mu} \quad (10)$$

$$\langle S,\mathcal{M}+\mu;\{-\mu\}|H_{eff}|S,\mathcal{M}-\mu;\{\mu\}\rangle = \frac{\hbar\Omega_0^2 N}{\delta}(A^N_{S,\mathcal{M}+\mu,\mu}A^N_{S,\mathcal{M}-\mu,-\mu}+B^N_{S,\mathcal{M}+\mu,\mu}B^N_{S,\mathcal{M}-\mu,-\mu}) \quad (11)$$

$$\langle S+2,\mathcal{M}+\mu;\{-\mu\}|H_{eff}|S,\mathcal{M}-\mu;\{\mu\}\rangle = \frac{\hbar\Omega_0^2 N}{\delta}A^N_{S,\mathcal{M}-\mu,-\mu}B^N_{S+2,\mathcal{M}+\mu,\mu} \quad (12)$$

$$\langle S-2,\mathcal{M}+\mu;\{-\mu\}|H_{eff}|S,\mathcal{M}-\mu;\{\mu\}\rangle = \frac{\hbar\Omega_0^2 N}{\delta}A^N_{S-2,\mathcal{M}+\mu,\mu}B^N_{S,\mathcal{M}-\mu,-\mu} \quad (13)$$

where $\{\mu = 1(-1)\}$ stand for optical state $|1,0\rangle$ ($|0,1\rangle$), $A^N_{S,M,\mu} = a^N_S C^{SM}_{S+1,1,M-\mu,\mu}$ and $B^N_{S,M,\mu} = b^N_S C^{SM}_{S-1,1,M-\mu,\mu}$ with $C^{S,M}_{S_1,S_2,M_1,M_2}$ being Clebsch-Gordan coefficient [16]. The eigenstate $|\mathcal{M},\nu\rangle$ is the dressed spinor BEC state.

III. MODIFIED SPIN MIXING

In experiment of spin mixing, the initial atomic state is a Fock state $|N_0,M\rangle$ that each spin component has fix number of atoms. N_0 is atomic number of spin component 0, $M\hbar$ is z direction angular momentum. Atomic number of spin component 1 and -1 can be expressed by N_0 and M. There are two processes that make the system evolute to the other Fock states. Process one is scattering between a pair of atoms, which can makes the spin component 1 and -1 jump to 0 and 0, and vice versa, due to spin dependent interaction. The system evolute to Fock state with the same M. Process two is absorbtion of a $\mu = 1(-1)$ photon and then emission of a $\mu = -1(1)$ photon by an atom. The system evolute to Fock state with $M \pm 2$, i.e. the z direction angular momentum of the spinor BEC increase(decrease) $2\hbar$.

Assume that the initial photon state is $|1,0\rangle$, i.e. the initial state of the system is $|N_0,M;1,0\rangle$. The time evolution of wave function is given by

$$\begin{aligned}
&|\Psi(t)\rangle = e^{-iH_{eff}t/\hbar}|N_0,M;1,0\rangle \\
&= \sum_{S',\mu}\sum_{\nu}\sum_{S}|S',\mathcal{M}-\mu;\{\mu\}\rangle\langle S',\mathcal{M}-\mu;\{\mu\}|e^{-iH_{eff}t/\hbar} \\
&|\mathcal{M},\nu\rangle\langle\mathcal{M},\nu|S,M;\{\mu_0\}\rangle\langle S,M;\{\mu_0\}|N_0,M;\{\mu_0\}\rangle_{\{\mu_0=1\}} \\
&= \sum_{S',\mu}|S',\mathcal{M}-\mu;\{\mu\}\rangle H^{S',\mu}_{N_0,\mathcal{M},\{\mu_0=1\}}(t) \qquad (14)
\end{aligned}$$

where $\mathcal{M} = M+1$. The matrix element between Fock state and total spin eigenstate $\langle S,\mathcal{M}-1|N_0,\mathcal{M}-1\rangle$ is given in reference [15]. After extracting the spin state of the Nth atom from the total spin eigenstate $|S',\mathcal{M}-\mu;\{\mu\}\rangle$ with equation (6), we can calculate the evolution of population of each spin component as

$$P_{\mu_0,\sigma} = P^{\mu=1}_{\mu_0,\sigma} + P^{\mu=-1}_{\mu_0,\sigma} \quad (15)$$

$$\begin{aligned}
P^{\mu}_{\mu_0,\sigma} \quad &= \sum_S[(A^N_{S,\mathcal{M}-\mu,\sigma})^2+(B^N_{S,\mathcal{M}-\mu,\sigma})^2]|H^{S,\mu}_{N_0,\mathcal{M},\{\mu_0=1\}}(t)|^2 \\
&+\sum_{S'}2A^N_{S'-1,\mathcal{M}-\mu,\sigma}B^N_{S'+1,\mathcal{M}-\mu,\sigma} \\
&Re\{H^{S'-1,\mu}_{N_0,\mathcal{M},\{\mu_0=1\}}(t)[H^{S'+1,\mu}_{N_0,\mathcal{M},\{\mu_0=1\}}(t)]^*\} \qquad (16)
\end{aligned}$$

where $N-S$ is even and $N-S'$ is odd. The probability of finding the photon in cavity mode $\{\mu\}$ is given as $P^{\mu}_{\mu_0} = \sum_{\sigma=\pm1,0}P^{\mu}_{\mu_0,\sigma}$. If the initial photon state is $|0,1\rangle$, the same equation with $\mathcal{M} = M-1$ and $\{\mu_0 = -1\}$ describes the time evolution. If the initial photon state stay in two cavity modes with the same probability, the evolution is given by $P_\sigma = (P_{\mu_0=1,\sigma}+P_{\mu_0=-1,\sigma})/2$. We can see that the time evolution is decided by symmetric of the atomic system as well as coupling strength and spin dependent interaction strength.

Numerical result of 300 Rb^{87} atoms with various initial condition is shown in figure 1. J is estimated by Thomas-Fermi approximation as $J = \hbar(\omega^2_{trap}/N)^{3/5}/X$ with $X = 1.52\times10^4$ and $\omega_{trap} = 514.5Hz$ being frequency of the optical trap [13], $\omega_e = 2\pi\times377.107THz = 2\omega_c$, and $\Omega_0 = 2\pi\times1MHz$ which is available in experiment [2]. The time is normalized by $t_{period} = \pi\hbar/J$, which is period of time evolution when there is no coupling between atoms and cavity. From figure 1 (a), comparison of (i) and (ii) shows that time evolution of coupling system has oscillation at time zone that non-coupling system is stationary. In figure (iii), the time evolution of probability of photon state at each polarization mode becomes random and fast oscillating around 0.5 after a short time of smooth relation. Changing to another initial condition in figure (b), time evolution of coupling and non-coupling system are almost the same. The photon state keeps oscillation between two polarization modes and relax to be stable. In figure (c), it was predicted in a former work that this kind of initial condition result in a long stable

evolution and then a short plus at time $t_{period}/8$. Figure (ii) shows that the coupling makes the stable evolution contains a small amplitude oscillation. The frequency of the oscillation small at early time and keep increasing, and then become a multi-plus at time $t_{period}/8$. In figure (iii), it is shown that the photon state oscillates at a very large frequency between two polarization modes with a quasi-constant amplitude.

IV. CONCLUSION AND DISCUSSION

We have model the coupling system of N Rb^{87} atoms and high Q optical cavity mode by two level approximation, i.e. only the transition $5^2S_{1/2}, F = 1 \longrightarrow 5^2P_{1/2}, F = 1$ of D line is consider. A more accurate model could be including other transition in D line of Rb^{87}. Because the frequency of optical cavity mode is far off resonance from transition of D line, the operator of excited state in the Hamiltonian can be removed, which result in an effective Hamiltonian. The effective Hamiltonian describe the coupling by two photons process, absorbtion of one photon from one cavity mode and then emission of one photon to another cavity mode. The eigenstate, or the dressed spinor BEC state, is obtained by diagonalizing the Hamiltonian under the basic set of direct product states, which are consist of total spin eigenstate and photonic Fock state of non-coupling Hamiltonian. The FPCs exhibit strong power in deducing the matrix elements of the Hamiltonian. With the dressed states in hand, the time evolution of wave function and average population in each spin component is derived. From the evolution equation (14) (15) and (16), when the system evolute to the direct product state with $\{\mu \neq \mu_0\}$, the photon evolute from the initial cavity mode to another cavity mode, and at the same time the spinor BEC evolute to the state with different z direction angular momentum. This process result in entanglement of the cavity modes and the spinor BEC. The numerical result show that the time evolution with some special initial condition is greatly modified by the cavity QED coupling. This effect is supposed to be observable experimentally. The time evolution with some other special initial condition, the oscillation of probability of finding a photon in each cavity mode is regular, which could be useful in quantum entanglement application or exploration.

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[1] A. Boca, R. Miller, K. M. Birnbaum, A. D. Boozer, J. McKeever, and H. J. Kimble, Phys. Rev. Lett. 93, 233603 (2004)

[2] P. Maunz, T. Puppe, I. Schuster, N. Syassen, P. W. H. Pinkse, and G. Rempe, Phys. Rev. Lett. 94, 033002 (2005)

[3] E.T.Jaynes and F.W.Cummings, Proc. IEEE 51, 89(1963).

[4] Peter Bermel, Alejandro Rodriguez, Steven G. Johnson, John D. Joannopoulos, and Marin Soljai, Phys. Rev. A 74, 043818 (2006),

[5] Michael Tavis and Frederick W. Cummings, Phys. Rev. 170, 379 - 384 (1968),

[6] Elena V. Goldstein, Ewan M. Wright, and P. Meystre, Phys. Rev. A 57, 1223(1998)

[7] E. S. Lee, C. Geckeler, J. Heurich, A. Gupta, Kit-Iu Cheong, S. Secrest, and P. Meystre, Phys. Rev. A 60, 4006(1999)

[8] J. Stenger, S. Inouye, D. M. Stamper-Kurn, H.-J. Miesner, A. P. Chikkatur, and W. Ketterle, Nature (London) 396, 345 (1998).

[9] Tin-Lun Ho, Phys. Rev. Lett. 81, 742 (1998).

[10] A. Sorensen, L.-M. Duan, J. I. Cirac, and P. Zoller, Nature (London) 409, 63 (2001).

[11] C. G. Bao and Z. B. Li, Phys. Rev. A 70, 043620 (2004).

[12] C. G. Bao and Z. B. Li, Phys. Rev. A 72, 043614 (2005).

[13] M. S. Chang, Q. Qin, W. X. Zhang, L. You, and M. S. Chapman, Nat. Phys. 1, 111 (2005), we use the data from the experiment as $\omega_{trap} = (\omega_x\omega_y\omega_z)^{1/3} = 514.5Hz$.

[14] C. K. Law, H. Pu, and N. P. Bigelow, Phys. Rev. Lett. 81, 5257 (1998).

[15] Ma Luo, Chengguang Bao, and Zhibing Li, Phys. Rev. A 77, 043625 (2008),

[16] D. A. Varshalovich, A. N. Moskalev, and V. K. Khersonskii, Quantum Theory of Angular Momentum (World Scientific, Singapore, 1988).

[17] Hans Kleinpoppen, "Atoms" in Ludwig Bergmann and Clemens Schaefer, Constituents of Matter: Atoms, molecules, Nuclei, and Particles, Wilhelm Raith, Ed. (Walter de Gruyter, Berlin, 1997)

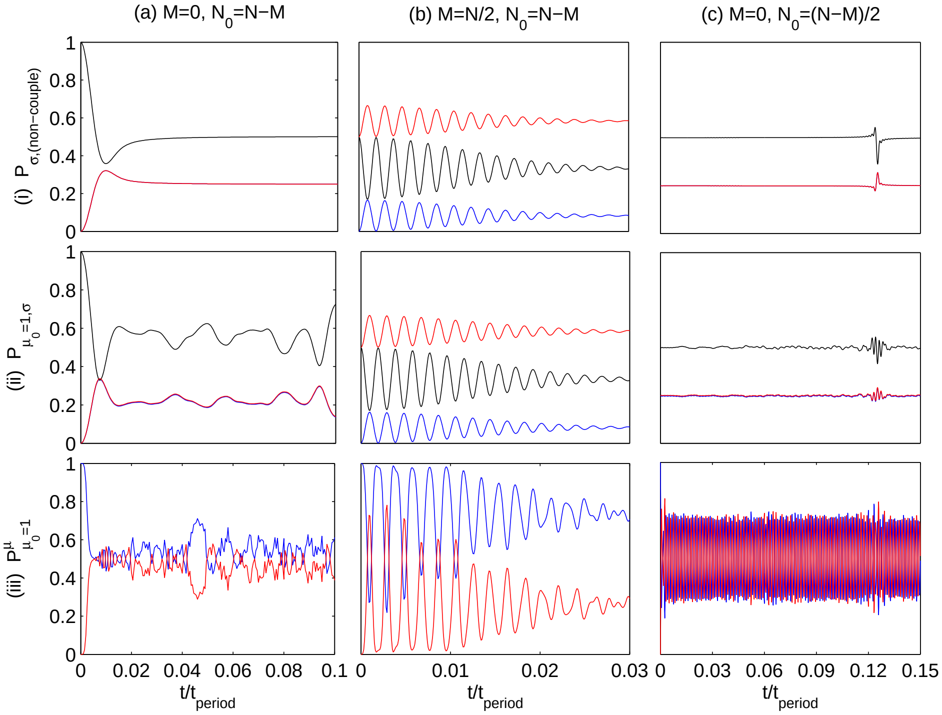

FIG. 1: Column (a) (b) and (c) is numerical result with different initial condition marked at the subtitle. Atomic number is $N = 300$. Row (i) and (ii) is time evolution of the average population at spin component $\sigma = 0$(black), 1(red), and -1(blue) without and with coupling between atoms and cavity, respectively. Row (iii) is time evolution of probability of the photon state at $\mu = 1$(blue) and -1(red) polarization mode.